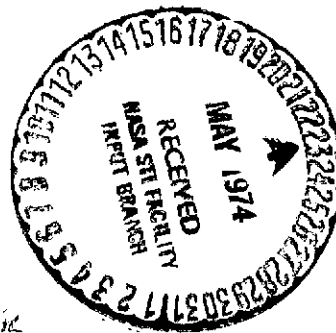


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RELIABILITY ANALYSIS
OF REDUNDANT SYSTEMS

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ABSTRACT

A method is proposed to compute the transition probability (the probability of partial or total failure) of parallel redundant system. An example is given to illustrate the simplification of this approach. The effects of geometry of the system, the direction of load, and the degree of redundancy on the probability of complete survival of parachute-like system are also studied. The results show that the probability of complete survival of three-member parachute-like system is very sensitive to the variation of horizontal angle of the load. However it becomes very insignificant as the degree of redundancy increases. Nevertheless, the three-member system is most reliable on the basis of equal volume of materials used under any load environments. As geometry is concerned, the system designed based on the minimum weight criterion is safer than those which have the same volume of materials and equal supporting members, except for certain cases where the angle between the load and the axis of the system gets larger.

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INTRODUCTION

The objective of structural engineers is to design such a structure that none of inadmissible states will occur during its service. Because of the uncertainties in load environments and structural strengthes, an ever-increasing effort has been made toward the application of probabilistic theory in structural design. In their pioneering work, Freudenthal, Garrelts, and Shinozuka (1)* have presented a rational approach to the structural reliability analysis which remains as the basis for all such analyses to-date. The state of the art of structural safety was recently summarized in a very comprehensive manner by Task Committee on Structural Safety of American Society of Civil Engineers (2).

With regard to redundant system, Hanai (3) discussed a three-member redundant truss with the only consideration of the structural strength being statistical in nature. Shinozuka and Yao (4,5) developed several methods to compute the transition probability for redundant systems basing on the assumption that the external load is equally distributed among the exsiting members. Later Yao and Yeh (6,7) proposed an approach to attack trusses by considering the tensile strength criterion and the

Numerals in parentheses refer to corresponding items in Appendix-- REFERENCES.

stress redistribution among the existing members according to geometrical position. The approach can also be extended to include the compressive failure criterion if the distribution function of structural compressive strength is known. Monte-Carlo technique was employed by Hoshiya (8) to obtain an approximate solution for the probability of failure of indeterminate structural system. Some of the difficulties encountered in the above-mentioned approaches are counting all possible failure paths. The computation will become very complicated if the number of degrees of redundancy increases.

The aim of this investigation is to systemize the computation methods of the probability of total or partial failure for parallel redundant systems. Furthermore, the effects of the direction of load, degree of redundancy, and system geometry on the reliability of parachute-like systems are discussed as well.

PARALLEL REDUNDANT SYSTEMS

Consider a structural system initially with m members as shown in Figure 1, and assume that the probabilistic function of resistant strength in each member is identical and is denoted by a random variable R with distribution function $F_R(x)$. Let W be the random variable of the applied load. The density function of W will be indicated by $f_W(x)$. Then the survival probability of k members among m initially existing members due to the statistical load W can be expressed as follows:

$$p_{mk} = \int_X p'_{mk}(x) f_W(x) dx \quad (1)$$

where $p'_{mk}(x)$ is the probability of survival of k members among m initially existing members due to a deterministic load w .

$p'_{mk}(x)$ can be computed from the following expression.

$$p'_{mk}(x) = T_{mk} [1 - F_R(x/k)]^k \quad (2)$$

where T_{mk} is the probability of failure of $(m-k)$ members among m initially existing members due to a deterministic load w .

The theorem of recurrence is used to compute T_{mk} as follows:

$$T_{mk} = T(m+1, m, m-1) T_{(m-1)k} + T'_{(m-1)k} \quad (3)$$

where

$$T(m-1, m-1-j, m-1-j-k) = \begin{cases} \binom{m}{k} [F_R(x/m)]^k & \text{if } 1 = j \\ \binom{m-1-j}{k} [F_R(\frac{x}{m-1-j}) - F_R(\frac{x}{m-1})]^k & \text{otherwise} \end{cases} \quad (4)$$

$\binom{m}{k}$ denotes the number of combination of "m" out of "k".

It is to be noted that $T_{(m-1)k} = 0$ if $k = m-1$ and $T(i,j,k)$ will be considered as $T(m,j,k)$ if i is larger than m . T_{mk} is the sum of function $T(i,j,k)$ and/or the product of functions $T(i,j,k)$. If T_{mk} only includes one T function, T'_{mk} can be obtained by adding one to the first two arguments of T function. In case T_{mk} has the product of T functions, T'_{mk} can also be derived from adding one to the first two arguments of T function which has the highest value of i in the sequence and adding one only to the first argument of T function which has the second highest value of i in the same sequence. For example:

If

$$T_{30} = T(4,3,2) [T(3,2,1) T(2,1,0) + T(3,2,0) + T(4,3,1) T(3,1,0) + T(4,3,0)] \quad (5)$$

then

$$T'_{30} = T(5,4,2) [T(4,2,1) T(2,1,0) + T(4,2,0) + T(5,4,1) T(4,1,0) + T(5,4,0)] \quad (6)$$

In order to perform the computation of p_{mk} , some assumptions should be made to the distribution functions of structural strengthes and applied loads. Let structural strength for each member in the systems considered be identical and follow a logarithmic normal distribution.

$$F_R(x) = \Phi \left[\frac{\log(x/\bar{R})}{s} \right] \quad (7)$$

The distribution of the load W is assumed to be second asymptotic distribution function of the largest values.

$$F_W(x) = \exp[-(x/W_c)^{-4.5}] \quad (8)$$

where $\Phi(x)$ is the standard normal distribution function, \bar{R} is the median, s is the standard deviation of structural strength, and W_c is the characteristic value of the load W so that $F_W(W_c) = 1/e$. The structural members are considered to be linearly elastic until failure caused by brittle fracture. Also the central factor of safety for the parallel system will be defined as $q = m\bar{R}/W_c$. With $q = 6$ and $s = 0.0416$ the numerical results of p_{mk} for this type of systems with $m = 3$ and $m = 10$ are computed and tabulated (see Table 1). The results show that probabilities of partial failure, i.e., p_{mk} , $m \neq k$ and $k \neq 0$ are very insignificant when compared with p_{mm} and p_{m0} . From these results, it can be said that if one member of a parallel redundant system fails the whole system will collapse.

PARACHUTE-LIKE SYSTEMS

Let C_i , L_i , and A_i respectively be the load coefficient, length, and cross-sectional area of the i th member in an axially symmetrical parachute-like system subjected to a statistical load W as shown in Figure 2. The probability of complete survival can be obtained from Equation (1).

$$p_{mm} = \int_X \prod_{i=1}^m [1 - F_R(C_i x)] f_W(x) dx \quad (9)$$

Since the sequence of failure among members will affect the probability of partial survival, p_{mk} , $k \neq m$, the approach for parallel system generally can not be applied to attack the parachute-like system. However it has been shown that the probability of partial failure is comparably small to the probability of total collapse, this study will emphasize p_{mm} or p_{m0} rather than p_{mk} , $m \neq k$. Though the accurate value of p_{m0} is difficult to obtain for large m because of counting process, the upper bound of p_{m0} can be calculated as follows:

$$p_{m0} \leq 1 - p_{mm} \quad (10)$$

Let q_i be the central factor of safety of i th member in parachute-like system with h ft in height and d ft in diameter (see Figure 2). q_i is defined in the following form:

$$q_i = \frac{\bar{R}}{C_i W_c} = \frac{A_i \bar{F}}{C_i W_c} \quad (11)$$

where \bar{r} is the median stress of materials. The cross-sectional area of i th member can be obtained from Equation (11).

$$A_i = q_i C_i W_c / \bar{r} \quad (12)$$

Then the total volume of a m -member parachute-like system is

$$V_m = (W_c / \bar{r}) q_i C_i L_i \quad (13)$$

If the system is subjected to the vertical load only; i.e., $u = 0$, where u is the angle between the load and the vertical axis of system, the coefficient of load factor becomes

$$C_i = \frac{1}{m \cos(z)} \quad (14)$$

where the angle z is defined as $\tan^{-1}(d/2h)$. Then Equation (13) becomes

$$V_m = \frac{W_c q d}{\bar{r} \sin(2z)} \quad (15)$$

In Equation (15) the subscript i in q_i is omitted since the central factor of safety is the same for each member. The minimum volume of such a system is obtained by letting $z = 45^\circ$ in Equation (15).

$$(V_m)_{\min} = W_c q' d / \bar{r} \quad (16)$$

where q' refers to the central factor of safety of system having minimum volume of materials used. It is interesting to

note that the volume of parachute-like system is independent of the number of members. Therefore the subscript m in V_m and $(V_m)_{\min}$ will be omitted.

The relative volume which is dimensionless is defined as the ratio of V and V_{\min} .

$$\frac{V}{V_{\min}} = \frac{q}{q' \sin(2z)} \quad (17)$$

Equation (17) shows that if two systems have the same volume, the central factor of safety will be related as $q = q' \sin(2z)$. On the other hand, same central factor of safety is used for both systems, the relative volume mainly depends on the geometry.

Parachute-like systems under the vertical statistical load are studied to determine the effect of geometry and degree of redundancy on the probability of complete survival. The results are computed from Equations (7), (8), and (9), and presented in Figures 3, 4, 5, and 6 by plotting the probability of survival versus the relative volume or the number of members. These results show that the probable safest system is designed on the basis of minimum weight criterion. They also show that the probability of complete survival decrease as the degree of redundancy of system increases.

Since most system will not only be subjected to vertical loads but also dominated by the action of horizontal force such

as wind force, the effects of the direction of inclined force, denoted by angles u and v , on the probability of complete survival are discussed as well in this study. The system considered here is the one which is designed to resist the vertical force only. The probability of complete survival for those systems with same volume subjected to inclined force are computed from Equations (7), (8), and (9), but the central factor of safety obtained from Equation (10) should be divided by the factor of $C_1 m \cos(z)$. The central factor of safety for each member in such a system is different from one another. The results are calculated as a function of the ratio of diameter and height, $d/2h$, and the number of members, m , are plotted in Figures 7 through 11. Since the load carried by each member is no longer uniformly distributed, the results as it expected show that the probability of complete survival decreases as the inclined angle u enlarges. Nevertheless, the system designed by using minimum weight criterion can not be said to be the safest system under such an environment as it can be noticeably in Figure 8, where system with $d/2h = 0.8$ is much safer than the system having minimum volume. From Figures 10 and 11, it can be noted that the horizontal direction of inclined force, v , has significant effect on the probability of survival for determinate system. The critical position of the inclined force is located at the

midsection of any two sequent members; i.e.,

$$v = (v_1 + v_{1-1})/2$$

However, the effect of variation of angle u on the survival probability becomes very insignificant as the degree of redundancy increases. Figure 9 explains that determinate parallel-like system is better than any indeterminate system.

CONCLUSIONS AND ACKNOWLEDGEMENTS

A method is presented to compute the probabilities of partial or total failure for the parallel redundant systems. Some examples are given to demonstrate the simplification of this method. As parachute-like systems are concerned, probabilities of complete survival are computed for determinate and indeterminate systems subjected to vertical or inclined force. The results show that the three-member parachute-like system is most reliable on the basis of equal volume of materials used under any load environments. It can also be concluded that the system with minimum weight performs best among all systems except for some cases where the angle between inclined load and the axis of system becomes larger. The only disadvantage of three-member system is more sensitive to the variation of the horizontal angle of the inclined load than that of multi-member systems.

This work was supported in part by National Aeronautics and Space Administration. The author wishes to thank Mr. A. E. Greaux, Dean of College of Engineering, Prairie View A&M University, for his support in this research.

TABLE 1.-- Results of Parallel Systems

$$m = 3, q = 6$$

| p_{33} | p_{32} | p_{31} | p_{30} |
|-----------|-------------------------|--------------------------|-------------------------|
| 0.9995241 | 3.9992×10^{-8} | 3.5219×10^{-20} | 4.6208×10^{-4} |

$$m = 10, q = 6$$

| p_{1010} | p_{109} | p_{108} | p_{107} |
|--------------------------|--------------------------|--------------------------|--------------------------|
| 0.9993679 | 1.17112×10^{-5} | 2.1729×10^{-7} | 1.8892×10^{-10} |
| p_{106} | p_{105} | p_{104} | p_{103} |
| 1.9839×10^{-15} | 3.1079×10^{-23} | 5.7726×10^{-35} | 0. |
| p_{102} | p_{101} | p_{100} | |
| 0. | 0. | 6.1026×10^{-4} | |

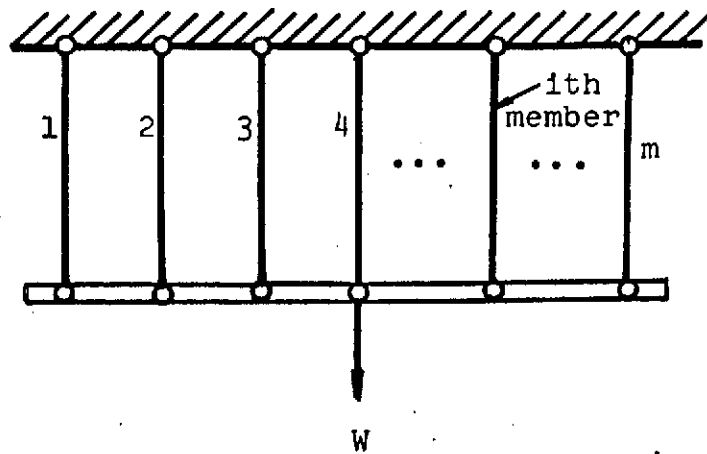


Figure 1.-- Redundant Parallel System

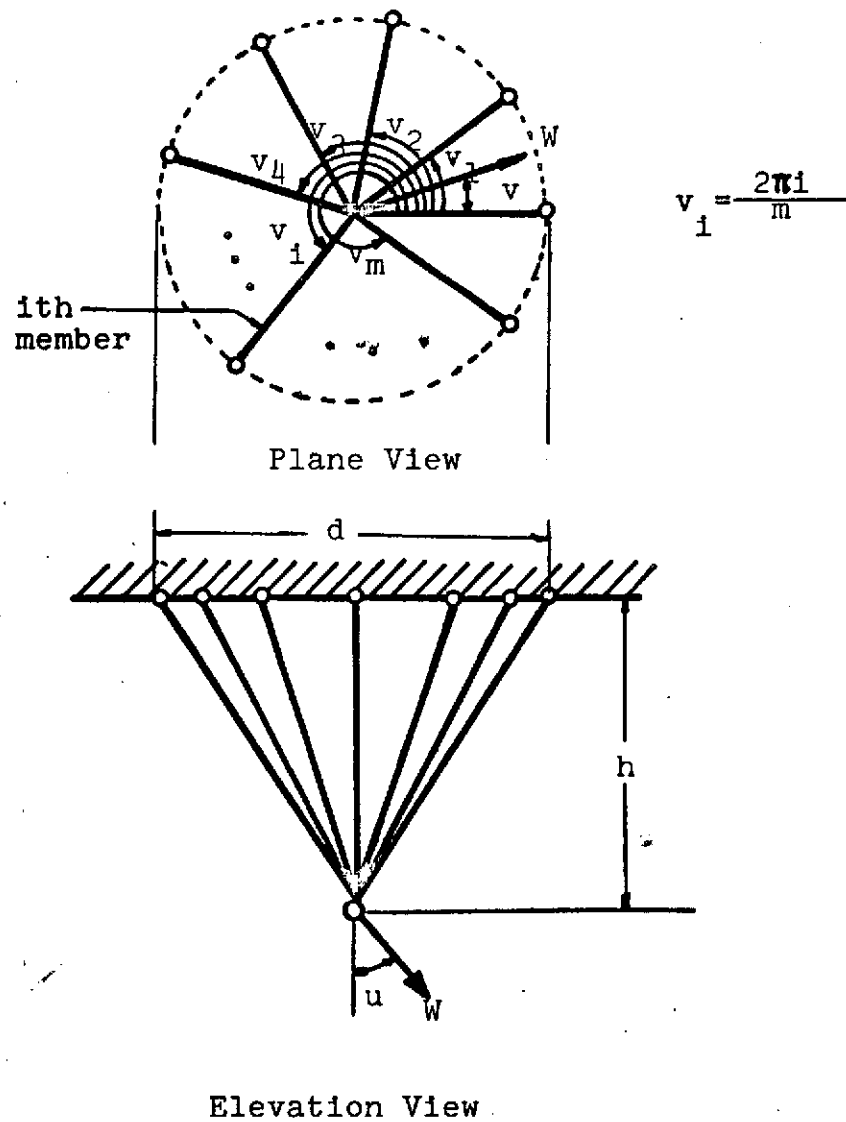


Figure 2.-- Parachute-like Axially Symmetrical System

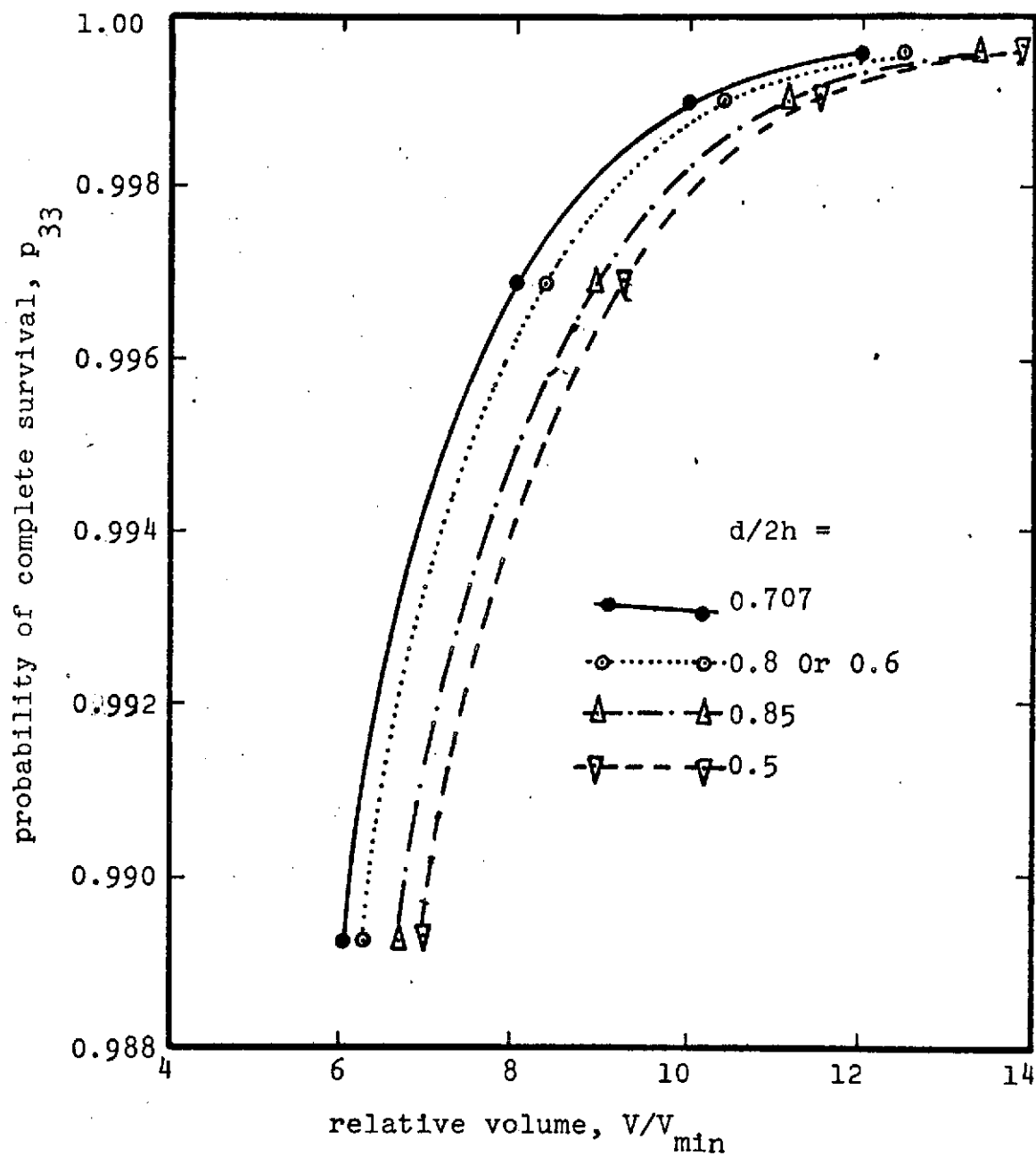


Figure 3.-- Probability of Complete survival of Three-member Parachute-like System ($u = 0$)

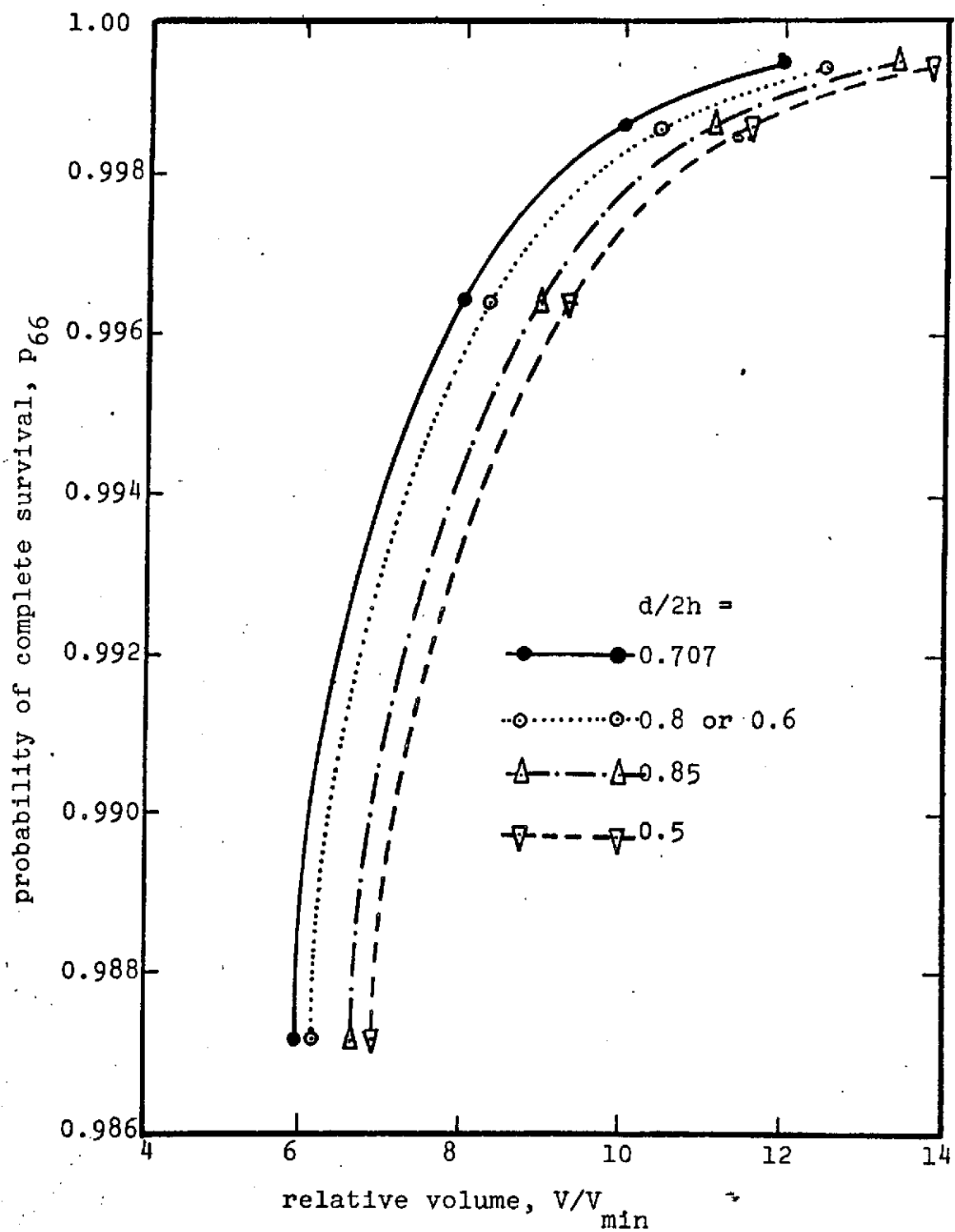


Figure 4.-- Probability of Complete Survival of Six-member Parachute-like System ($u = 0$)

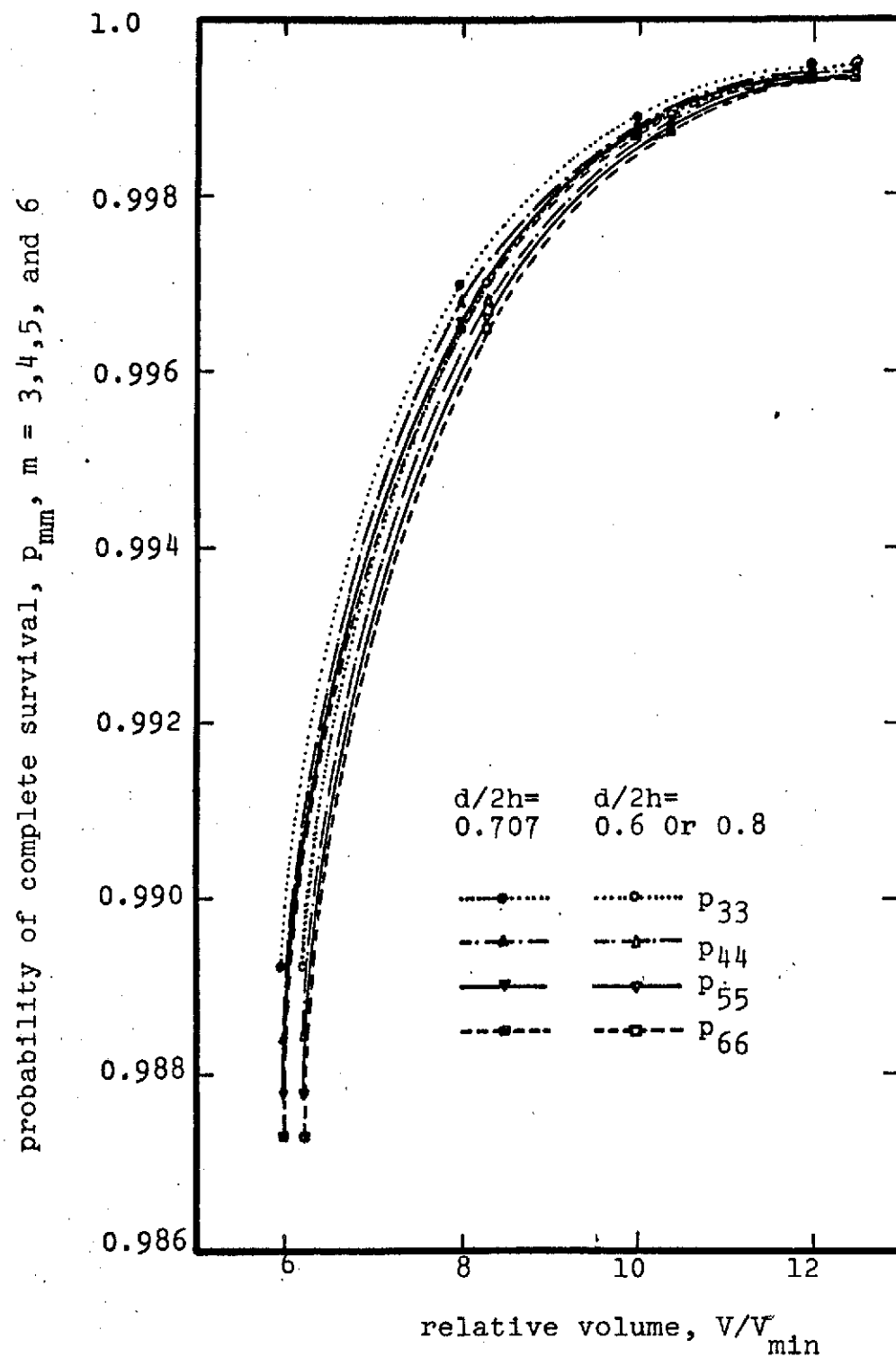


Figure 5.-- The Effect of Number of Degrees of Redundancy on the Probability of Complete Survival ($u = 0$)

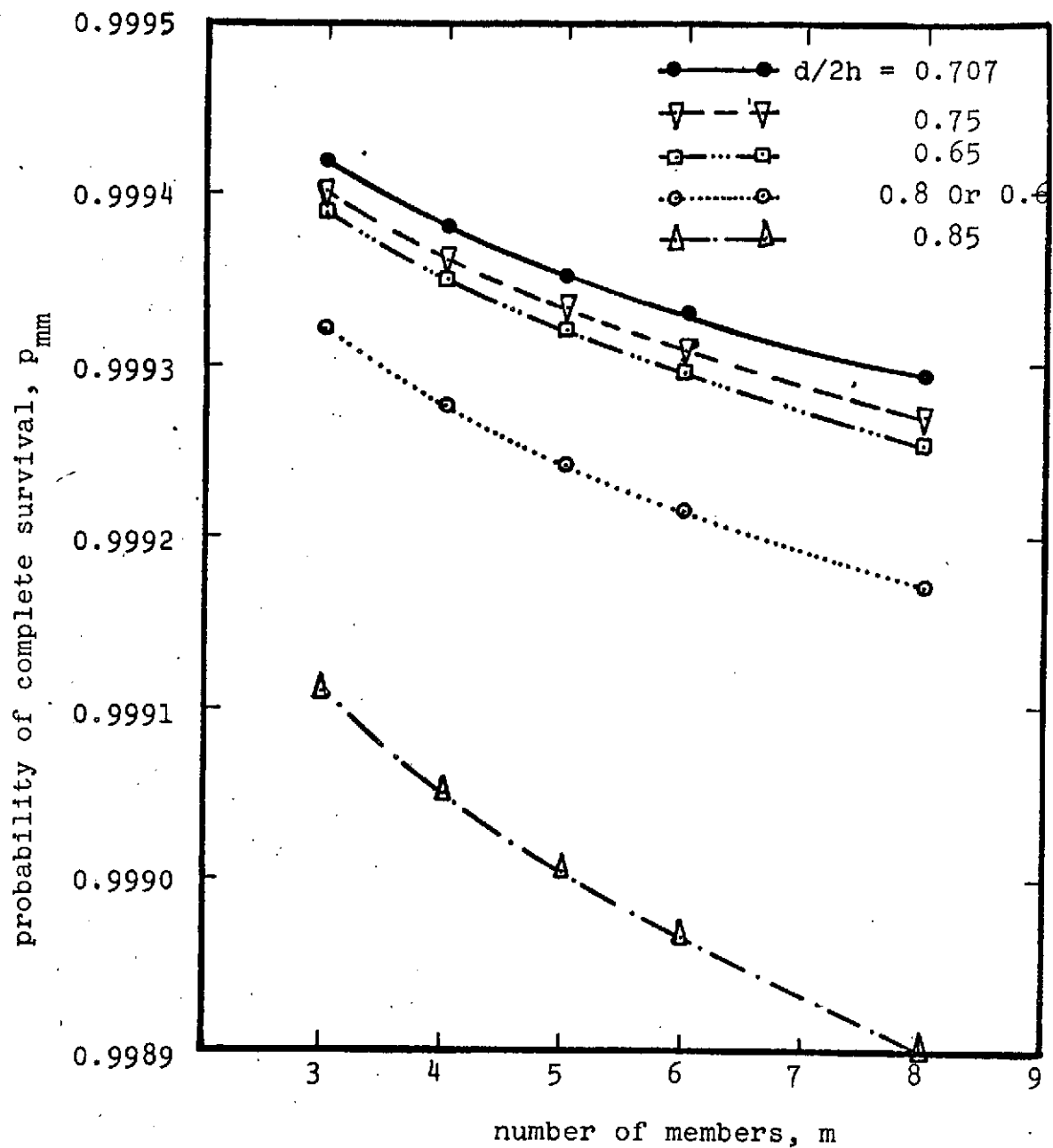


Figure 6.-- The Effect of System Geometry and Number of Degrees of Redundancy on the Probability of Complete Survival ($u = 0$)

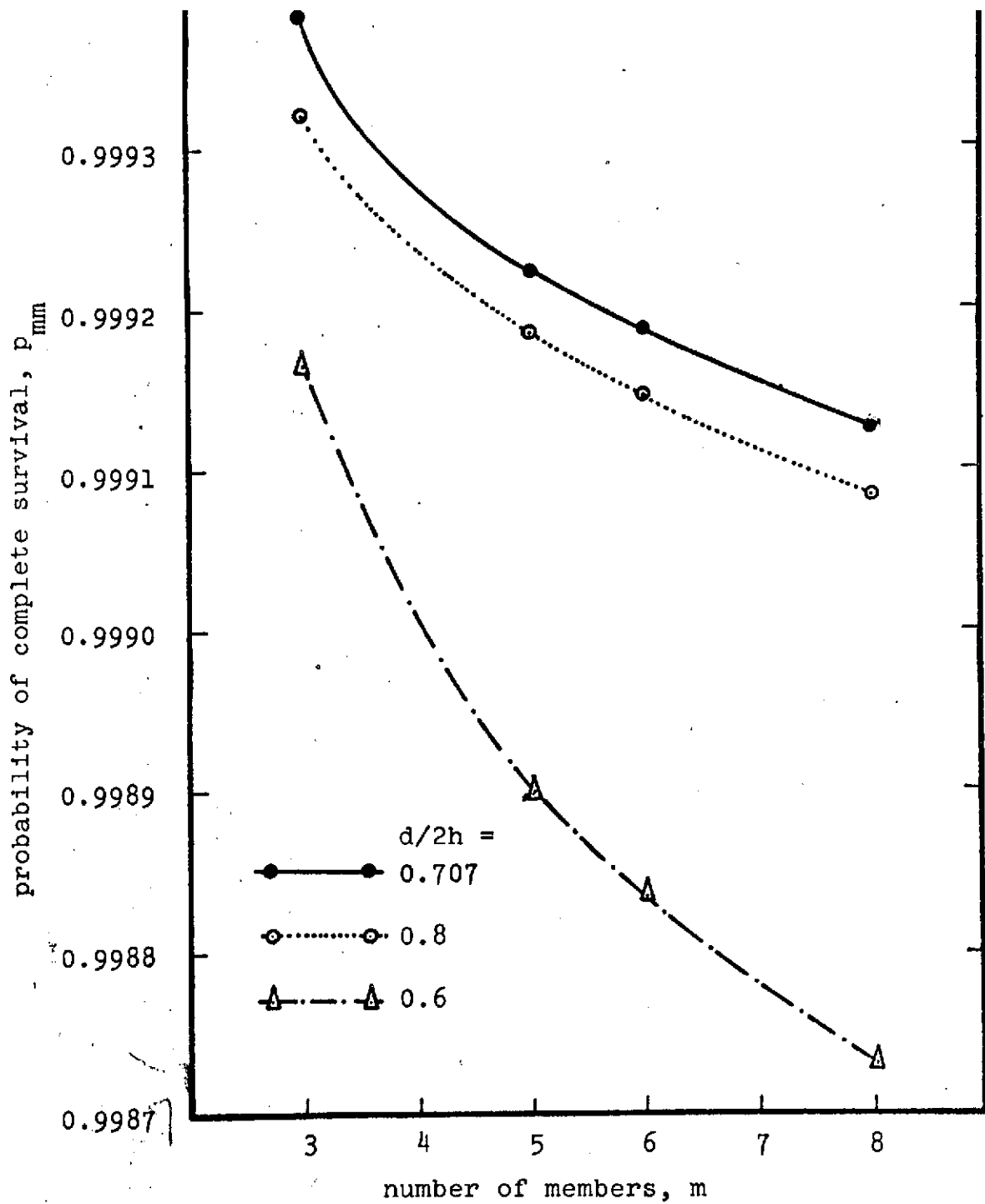


Figure 7.-- The Effect of System Geometry and Number of Degrees of Redundancy on the Probability of Complete Survival ($u = 5^\circ$ and $v = 0$)

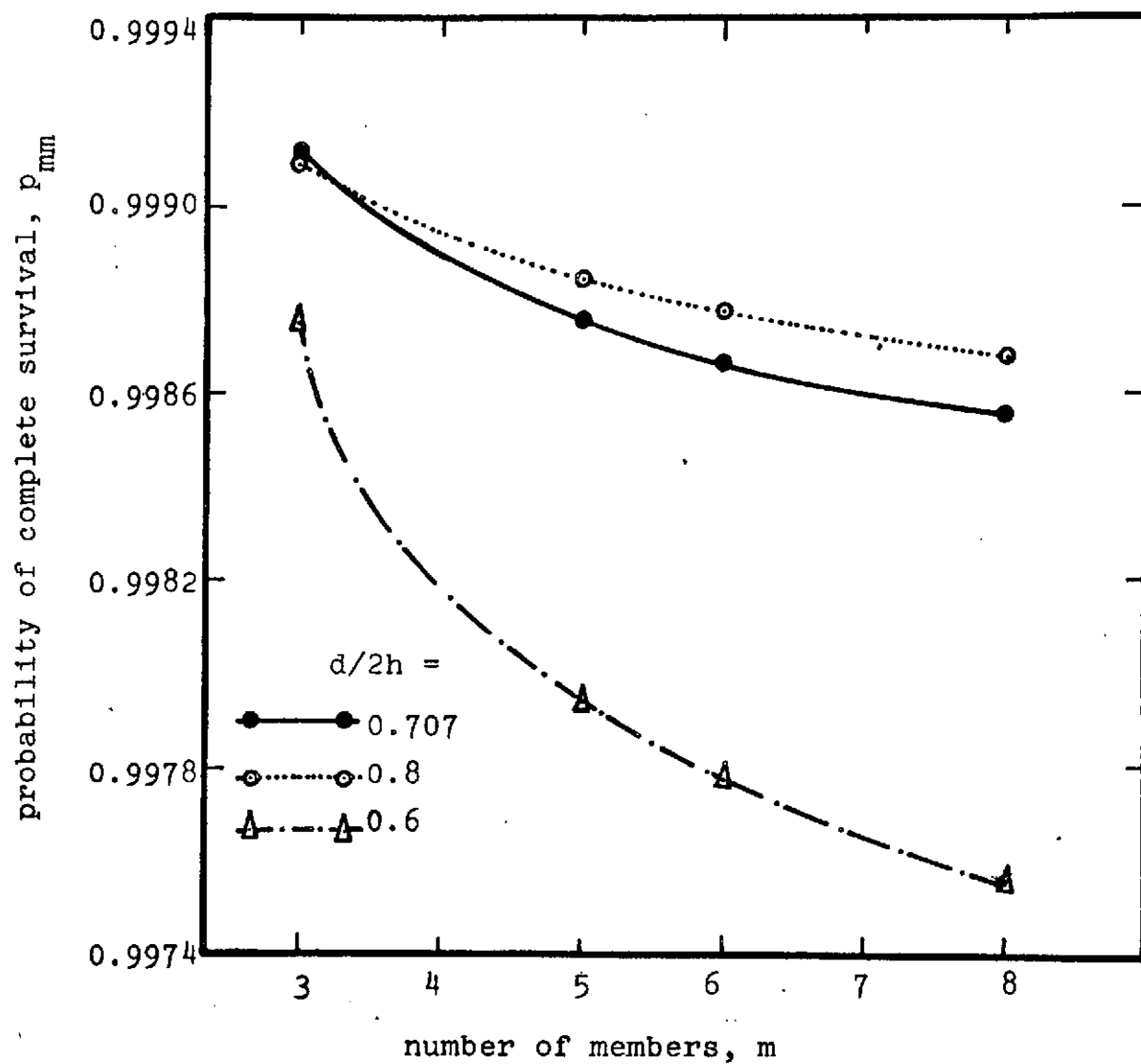


Figure 8.-- The Effect of System Geometry and Number of Degrees of Redundancy on the Probability of Complete Survival ($u = 10^\circ$ and $v = 0$)

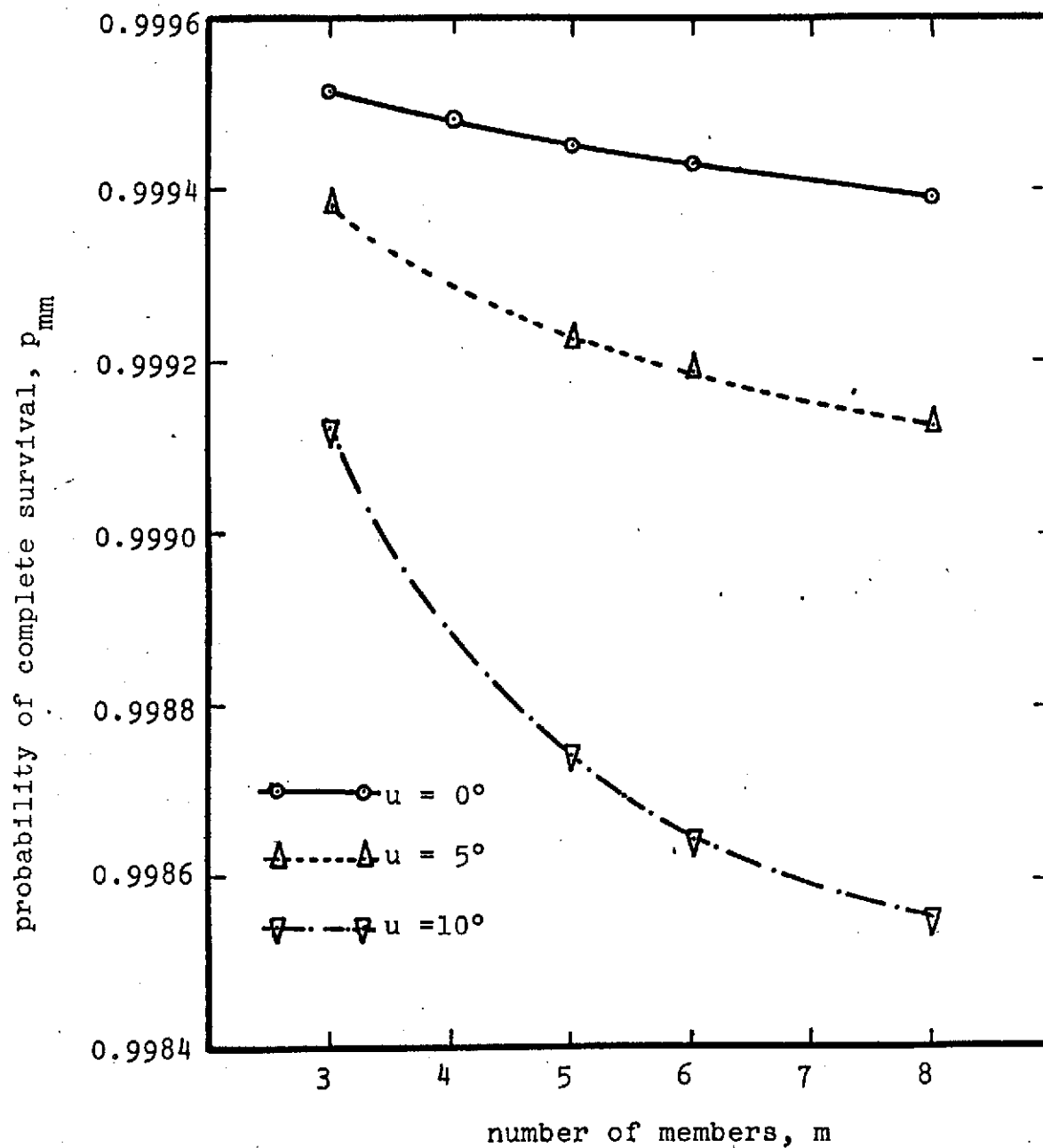


Figure 9.-- The Effect of Variation of Vertical Angle, u , on the Probability of Complete Survival of the System with $d/2h = 0.707$ ($v = 0$)

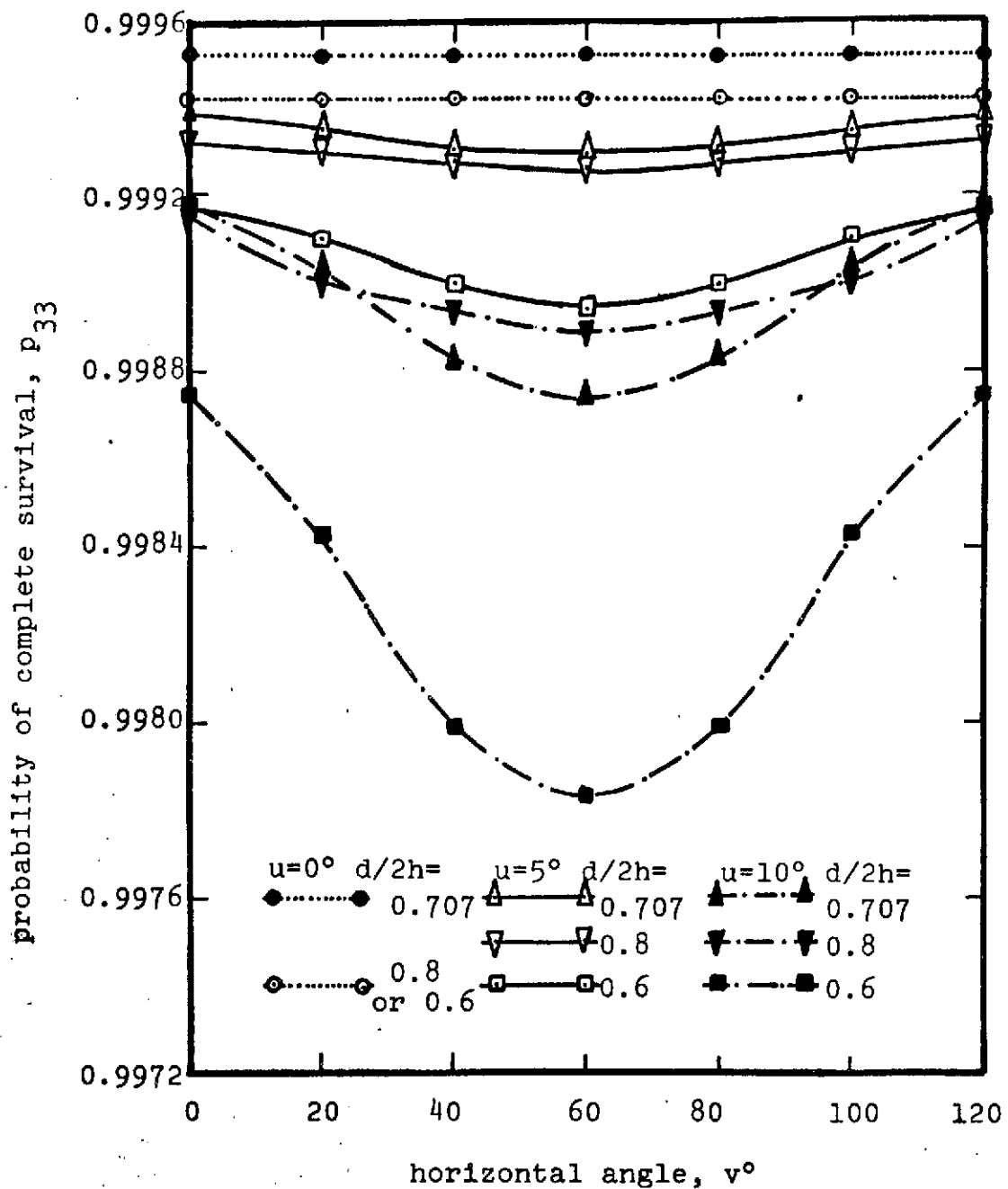


Figure 10.-- Probability of Complete Survival of Three-member Parachute-like System due to the Variation of Horizontal Angle, v

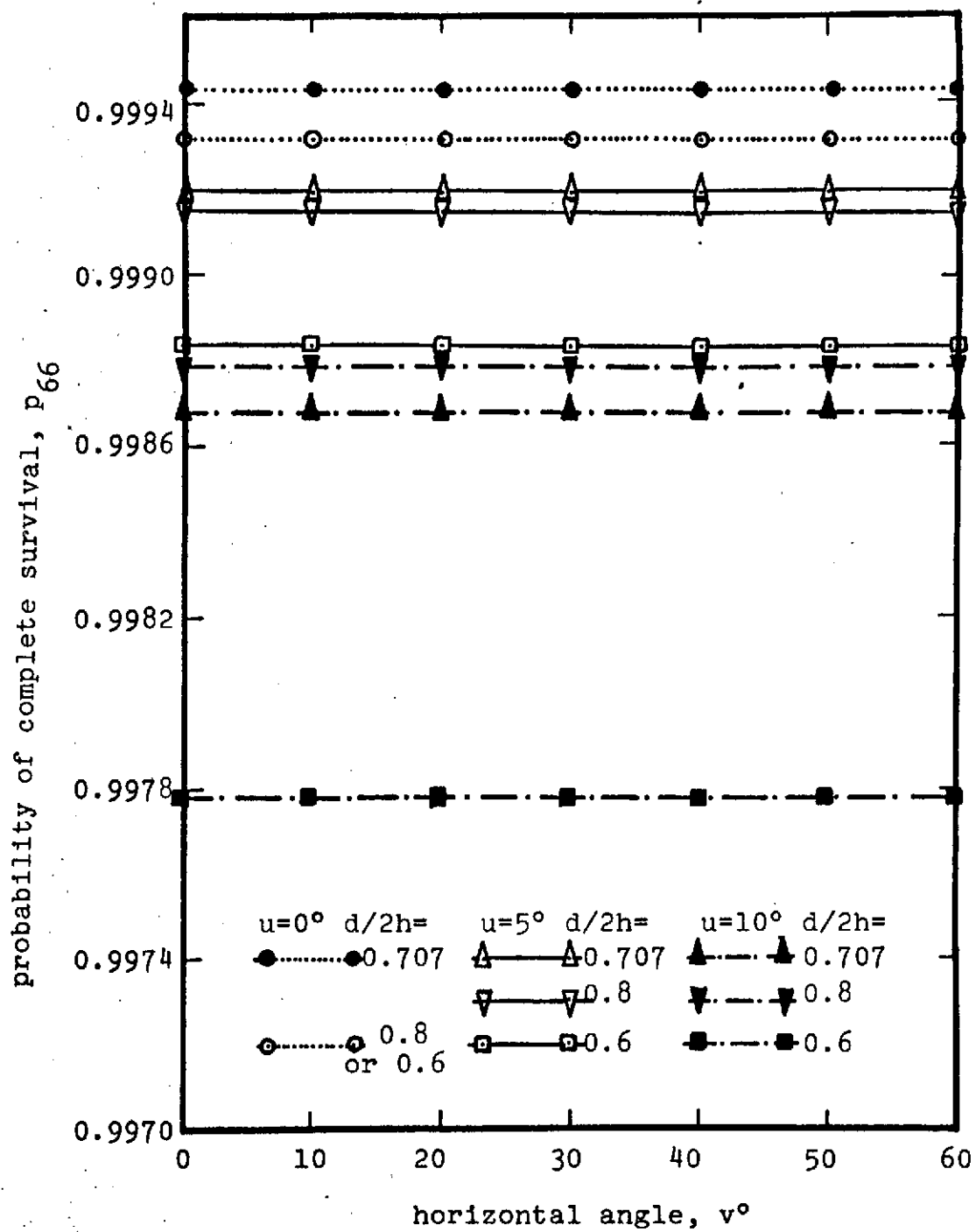


Figure 11.-- Probability of Complete Survival of Six-member Parachute-like System due to the Variation of Horizontal Angle, v

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